Cusp Surface Analysis, version of June 19, 1992.

Model: 0 = Alpha + Beta*(Y–Gamma) – Delta*(Y–Gamma)^3.

The conditional density of Y given X[1],...,X[v]:

f(Y|X) = exp[Psi + Alpha*Z + Beta*Z^2/2 – Delta*Z^4/4],

where Z = Y – Gamma,
Psi = constant (with respect to Y),
Alpha = A[0] + A[1]*X[1] + ... + A[v]*X[v],
Beta = B[0] + B[1]*X[1] + ... + B[v]*X[v],
Gamma = C[0] + C[1]*X[1] + ... + C[v]*X[v],

and v = 2 (in this analysis).

Maximum Likelihood Estimation for the Cusp Model:

Cases = 33
Log–Likelihood = –33.3502

Standard coefficients, with t–statistics in parentheses:

Var	Alpha	Beta	Gamma	Delta
Const	0.131 (0.5)	3.326 (2.2)	-0.146 (-1.5)	3.071 (2.7)
2	0.438 (1.5)	-0.318 (-0.4)	-0.304 (-2.4)	
3	-0.313 (-1.3)	-1.829 (-2.1)	0.036 (0.4)	

(Each t-statistic has 24 degrees of freedom)

Raw coefficients:

Var	Alpha	Beta	Gamma	Delta
Const	8.517e-3	1.151e-2	5.202e+1	6.981e-6
2	2.897e-2	-8.159e-4	-1.331e+1	
3	-4.306e-4	-9.768e-5	3.308e-2	

**Test for H0: Conditional densities are Type N1 (linear regression)
versus H1: Conditional densities are Type N3 (cusp regression)**

>>>>>> Asymptotic Chi-square = 23.66 (df = 6) <<<<<<

**Test for H0: Delta = 0 (i.e. no cubic term)
versus H1: Delta > 0 (a one-tailed test)**

>>>>>> t = 2.65 (df = 24) <<<<<<

Estimated correlations between Alpha and Gamma estimators:

	G0	G1	G2
A0	-0.48	0.27	-0.07
A1	0.19	-0.39	-0.28
A2	-0.20	-0.19	-0.53

Predictions based on this analysis: |<------- D A T A ------->|

Case	Asymm	Bifur	Mode	Antimode	Mode	Y(# 1)	X(# 2)	X(# 3)
1	0.68	0.96			49.41	35.00	1.91	87.00
2	-0.73	3.81	27.85	63.83	84.51	38.00	-0.35	62.00
3	0.30	4.23	13.76	41.20	74.11	74.00	0.79	46.00
4	0.26	1.05			54.06	53.00	1.39	90.00
5	-0.15	2.22	23.06	47.60	66.78	41.00	0.66	78.00
6	-0.20	1.38	25.74	48.47	59.63	35.00	0.78	90.00
7	0.11	4.36	15.99	45.69	77.37	53.00	0.54	46.00
8	0.43	5.30	10.57	41.19	78.12	6.00	0.73	30.00
9	0.24	4.27	14.47	42.64	75.15	74.00	0.71	46.00
10	0.32	1.01			53.23	65.00	1.47	90.00
11	-0.36	2.96	24.14	54.08	74.39	62.00	0.26	70.00
12	-0.89	2.54	31.39	71.76	72.12	26.00	-0.28	81.00
13	0.12	7.20	13.44	52.22	92.29	22.00	-0.03	7.00
14	0.54	0.85			50.00	10.00	1.77	90.00
15	0.05	1.42	23.67	39.96	58.72	74.00	1.06	87.00
16	0.44	0.92			51.53	10.00	1.63	90.00
17	0.34	4.19	13.24	40.10	73.32	10.00	0.85	46.00
18	0.58	2.71	14.28	29.06	61.56	53.00	1.44	64.00
19	-0.19	1.59	25.02	48.01	61.71	35.00	0.75	87.00
20	0.08	7.22	13.96	53.02	92.96	86.00	-0.08	7.00
21	0.34	4.20	13.33	40.29	73.46	62.00	0.84	46.00
22	0.22	4.29	14.73	43.18	75.54	62.00	0.68	46.00
23	-0.23	2.28	23.77	49.76	67.90	74.00	0.56	78.00
24	0.21	6.91	12.25	49.69	89.51	94.00	0.14	10.00
25	0.57	6.65	7.35	41.88	83.06	94.00	0.62	10.00
26	-0.38	2.39	25.37	54.23	70.21	26.00	0.35	78.00
27	0.40	3.57	13.66	36.88	68.94	10.00	1.05	54.00
28	0.37	2.87	15.80	35.46	65.21	35.00	1.15	64.00
29	0.51	6.91	7.96	43.67	85.15	74.00	0.50	7.00
30	-0.34	5.27	21.14	57.32	88.56	14.00	-0.18	38.00
31	0.03	2.23	21.04	42.44	64.97	71.00	0.88	76.00
32	0.41	0.94			51.96	53.00	1.59	90.00
33	0.24	1.06			54.27	35.00	1.37	90.00

Location of data in the control space:

Vertical axis: Bifurcation (splitting) factor
Horizontal axis: Asymmetry (normal) factor
Asterisks: Bimodal zone

```
       -5    -4    -3    -2    -1     0     1     2     3     4     5
      ***************************************************
 5.0    0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0    5.0
         *********************************************
 4.5    0  0  0  0  0  0  0  0  0  0  3  0  0  0  0  0  0  0  0  0  0    4.5
           ***************************************
 4.0    0  0  0  0  0  0  0  0  0  1  0  3  0  0  0  0  0  0  0  0  0    4.0
             *********************************
 3.5    0  0  0  0  0  0  0  0  0  0  0  1  0  0  0  0  0  0  0  0  0    3.5
               ***************************
 3.0    0  0  0  0  0  0  0  0  0  1  0  1  0  0  0  0  0  0  0  0  0    3.0
                 ********************
 2.5    0  0  0  0  0  0  0  0  1  1  1  1  0  0  0  0  0  0  0  0  0    2.5
                   ***************
 2.0    0  0  0  0  0  0  0  0  0  0  2  0  0  0  0  0  0  0  0  0  0    2.0
                     ***********
 1.5    0  0  0  0  0  0  0  0  0  0  3  0  0  0  0  0  0  0  0  0  0    1.5
                       *******
 1.0    0  0  0  0  0  0  0  0  0  0  1  6  0  0  0  0  0  0  0  0  0    1.0
                         ***
 0.5    0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0    0.5
                         *
 0.0    0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0    0.0

-0.5    0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0   -0.5

-1.0    0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0   -1.0

-1.5    0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0   -1.5

-2.0    0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0   -2.0

-2.5    0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0   -2.5

-3.0    0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0   -3.0

-3.5    0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0   -3.5

-4.0    0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0   -4.0

-4.5    0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0   -4.5

-5.0    0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0   -5.0

       -5    -4    -3    -2    -1     0     1     2     3     4     5
```

7 cases did not fit in the above figure.

Linear R^2 = 0.095 (Multiple regression)
Delay R^2 = 0.695 (Attracting-mode convention)
Maxwell R^2 = -0.178* (Most-likely-mode convention)

* Negative R^2 values occur when the cusp model is worse a constant model.

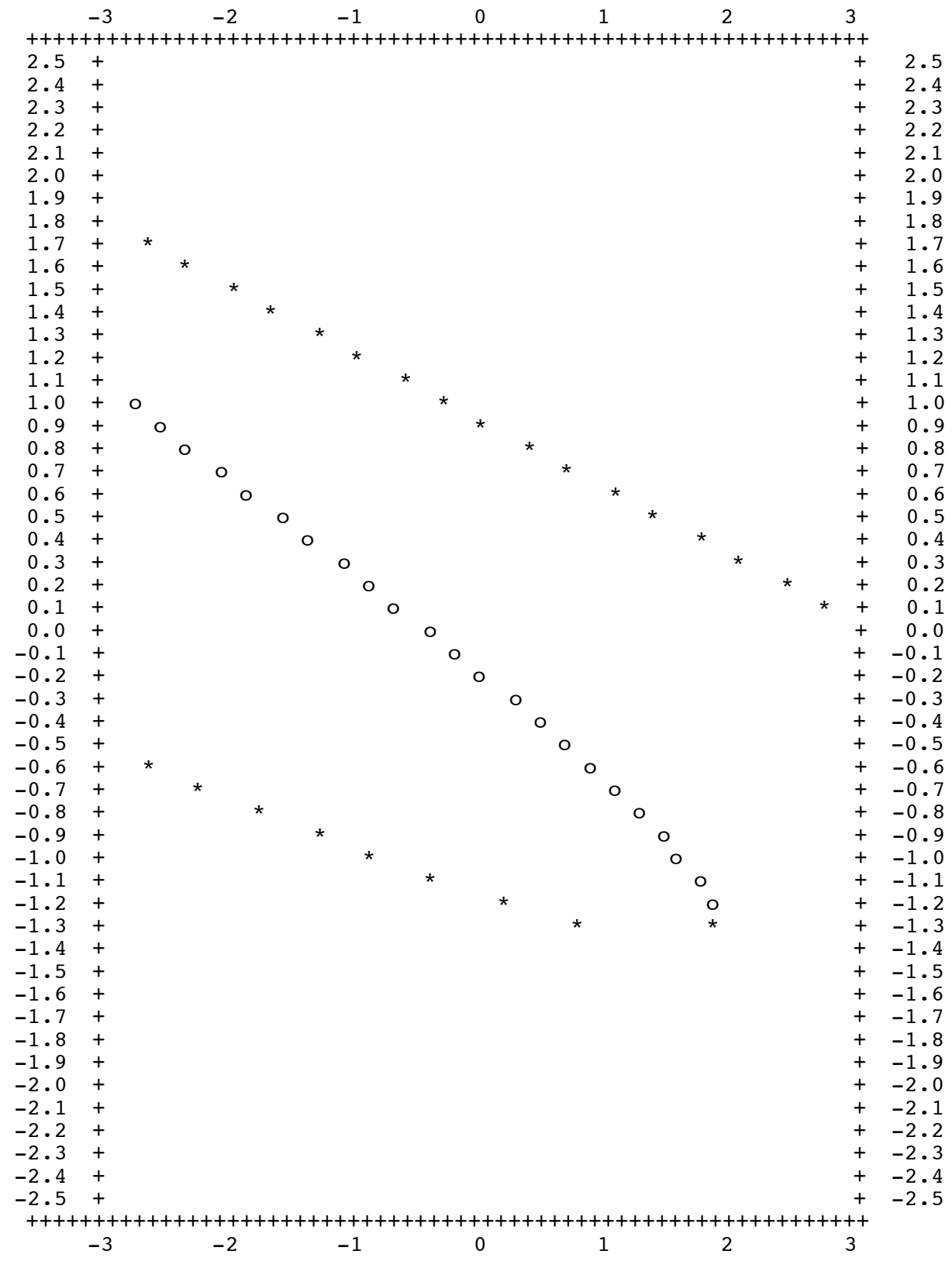
>>>>>> Fraction of cases in bimodal zone: 0.788 <<<<<<

Histogram of residuals from predictions of the delay rule:
(Units are standard deviations of the dependent variable.)

Y	N	Frequency
-3.00	0	|
-2.75	0	|
-2.50	0	|
-2.25	0	|
-2.00	0	|
-1.75	0	|
-1.50	2	|##
-1.25	0	|
-1.00	1	|#
-0.75	1	|#
-0.50	5	|#####
-0.25	7	|#######
0.00	5	|#####
0.25	5	|#####
0.50	5	|#####
0.75	2	|##
1.00	0	|
1.25	0	|
1.50	0	|
1.75	0	|
2.00	0	|
2.25	0	|
2.50	0	|
2.75	0	|
3.00	0	|

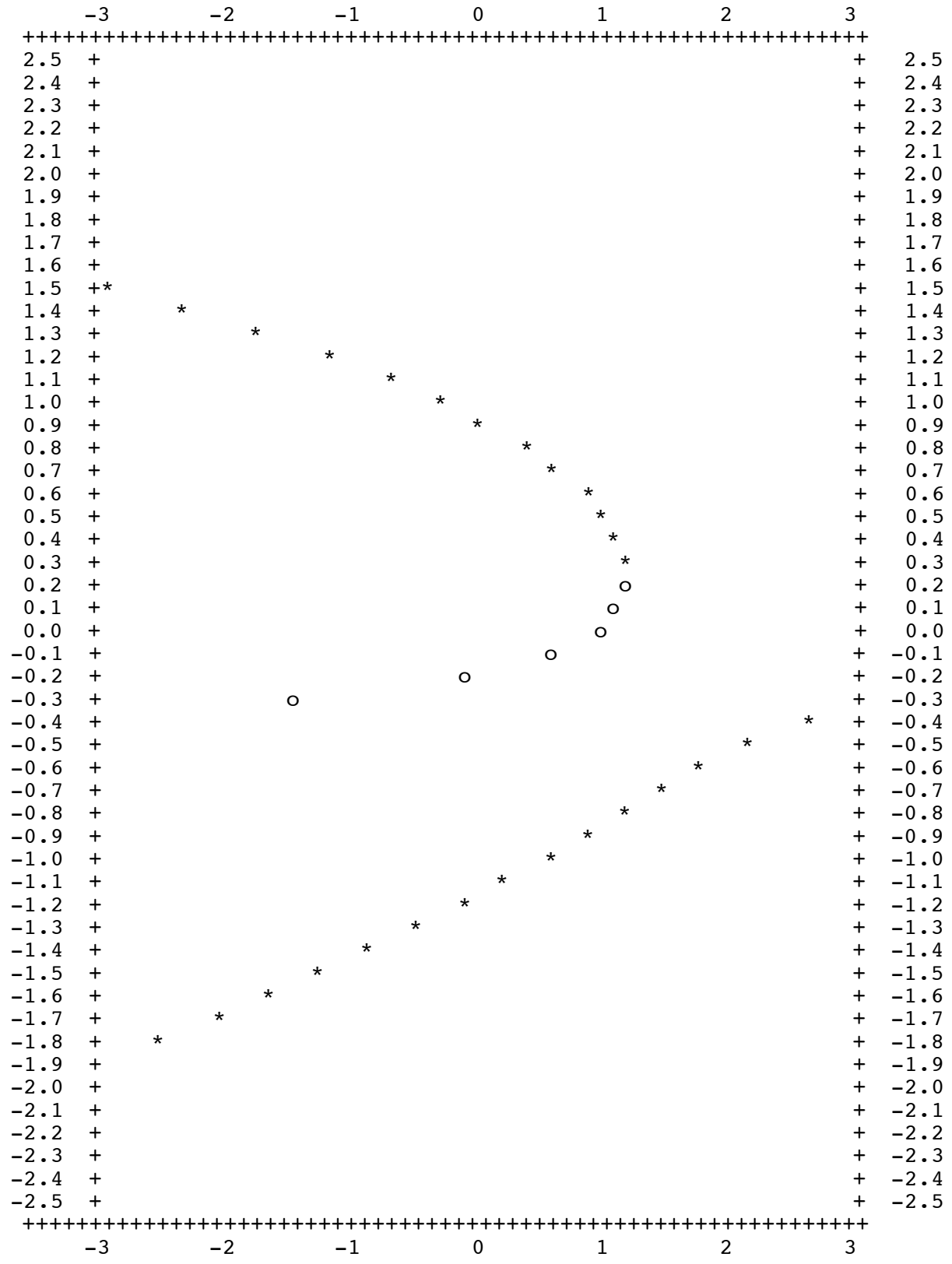
Error Mean = -0.116
Error St.Dev = 0.553

Effect of variable 2, holding all others constant at their mean values.



* Mode symbol
o Antimode symbol

Effect of variable 3, holding all others constant at their mean values.



* Mode symbol
o Antimode symbol